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Massless Localized Vector Field on a Wall in $D = 5$ SQED with Tensor Multiplets

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Abstract

Massless localized vector field is obtained in five-dimensional supersymmetric (SUSY) QED coupled to tensor multiplets as a half BPS solution. The four-dimensional gauge coupling is obtained as a topological charge. We also find all the (bosonic) massive modes exactly for a particular value of a parameter, demonstrating explicitly the existence of a mass gap. The four-dimensional Coulomb law is shown to hold for sources placed on the wall.

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1 Introduction

Brane-world scenario has raised a new possibility to obtain unified theories beyond the standard model [1, 2, 3]. To realize the brane-world scenario, it is necessary to localize standard model particles on topological defects such as domain walls. It has been a long-standing problem to obtain a massless vector field localized on a wall. If we implement the Higgs mechanism in the bulk and restore the gauge symmetry on the wall, we can in fact localize the vector field on the wall. However, it has been pointed out that superconducting bulk will absorb flux coming out of the source placed on the wall. Therefore these flux will not reach beyond the width of the wall even in the direction along the world volume of the wall. This screening implies that the vector field should have a mass of the order of the inverse width of the wall [4, 2]. This general argument are confirmed by explicit examples such as [5, 6]. Then one is naturally led to consider a dual picture as an appropriate setting. If a vector field is confined in the bulk and deconfined on the wall, the flux coming out of a source should be repelled from the wall, producing a four-dimensional Coulomb law in the world volume of the wall. This mechanism of massless localized vector field has been argued, and a toy model in four-dimensions has been proposed [4]. This is a nice general idea, but a concrete model had to use the nonperturbative effect to confine vector field, which is not at all obvious to work in higher dimensions such as five dimensions. Another mechanism that has been proposed was to use gravity. It has been shown that vortex together with the warp factor of gravity are needed to obtain a massless localized vector field [7, 9, 8]. It is perhaps more desirable to obtain a model which gives a massless localized vector field even in the limit of vanishing gravitational coupling, since the gravitational effects are known to be small. Another model [10] generalized the idea of induced gauge field by quantum effects [11]. This idea is old [12] and attractive, but is rather difficult to obtain a reliable approximation scheme for such a quantum effects.

Taking a massive $\mathcal{N}=2$ SUSY QED in four-dimensions as a toy model, it has been argued that a massless localized gauge field W_μ is obtained by dualizing the massless Nambu-Goldstone scalar ϕ in the three-dimensional effective theory $2\partial_{[\mu}W_{\nu]} = \epsilon_{\mu\nu\rho}\partial^\rho\phi$ [13] [1]. This is certainly an intriguing result, but is difficult to generalize to our realistic situation of higher dimensions, since the gauge field is dual of a compact scalar only in three-dimensional space-time. In five dimensions[2], a straightforward application of the electromagnetic duality for a vector field W_M should give a tensor (two form) field B_{MN}

$$F_{MNL}(B) = \frac{1}{2}\epsilon_{MNLPQ}F^{PQ}(W), \tag{1.1}$$

where the field strengths are defined by

$$F_{MNL}(B) \equiv 3\partial_{[M}B_{NL]}, \quad F_{MN}(W) \equiv 2\partial_{[M}W_{N]}. \tag{1.2}$$

An interesting approach has been proposed using tensor field to obtain a massless localized vector field [14]. They assumed that a tensor field in five dimensions couples to some physically motivated wall configuration which is given and fixed externally, and argued for a (quasi-)localization and the charge universality in generic terms.

[1] Antisymmetrization of indices are denoted by brackets. We use the convention to divide by the number of terms in the antisymmetrization such as $\partial_{[M}W_{N]} \equiv (\partial_M W_N - \partial_N W_M)/2$.

[2] We will denote the five-dimensional indices by capital Latin characters $M, N = 0, 1, \cdots, 4$ and four-dimensional indices by Greek characters $\mu, \nu = 0, 1, 2, 3$.

The purpose of our paper is to give a concrete supersymmetry (SUSY) model (with eight supercharges) including tensor multiplets together with a vector and hypermultiplets in five dimensions, in order to construct a fully consistent model for a massless localized vector field. We find a wall and a massless vector multiplet localized on the wall as a consistent solution of the equations of motion. Since our wall configuration preserves half of SUSY [6], we obtain a Massless $U(1)$ vector multiplet in the $\mathcal{N}=1$ SUSY four-dimensional effective theory. We find that the four-dimensional gauge coupling is expressed as a topological charge associated with the wall. Moreover, we obtain not only the massless mode but also massive modes of the vector multiplet exactly in one choice of a parameter. By introducing a static source, we show that the four-dimensional Coulomb law of the usual minimal electromagnetic interaction is reproduced. Our mechanism has some similarities to that in Ref.[14], such as the generic nature of the mechanism of the massless localized vector multiplet. However, we have a fully consistent model including the wall and the massless localized vector field as solutions of equations of motion, without an ad hoc assumption for the wall as a given external configuration. Moreover we start from a SUSY theory in five dimensions, resulting in an $\mathcal{N}=1$ SUSY effective low-energy theory.

In Sect.2, our model with the tensor multiplet is introduced. In Sect.3, massless localized vector field is obtained. All the massive modes are also found for a particular value of a parameter. An effective Lagrangian containing all massive modes is also worked out to the quadratic order. In Sect.4, the four-dimensional Coulomb law is obtained between sources placed on the wall. In Sect.5, a possible generalization (without SUSY) to arbitrary space-time dimensions is proposed and a number of remaining issues are noted. Some details of massive mode functions are given in Appendix.

2 Our Model with Tensor Multiplets

It has been known that tensor multiplets can couple to vector multiplets in five-dimensional SUSY theories. On the other hand, a five-dimensional SUSY model with hypermultiplets coupled to a $U(1)$ vector multiplet can give a domain wall as a half BPS solution, producing a wall configuration for the vector multiplet scalar Σ [6]. To build a wall, we introduce a $U(1)$ vector multiplet, whose bosonic components are gauge field W_M, scalar field Σ, and $SU(2)_R$ triplet of auxiliary fields $Y^a, a=1,2,3$. We also need hypermultiplets, whose bosonic components are $SU(2)$ doublets of scalar fields H^{iA}, and auxiliary fields F_i^A, with the $i=1,2$ and A is the flavor indices of hypermultiplets. The number of SUSY vacua is equal or less than the number of hypermultiplets. To obtain a single wall solution, we take the number of hypermultiplets to be two, for simplicity : $A=1,2$. It has been shown that these two types of multiplets suffice to produce a wall in five dimensions preserving half of SUSY (a 1/2 BPS state) [6]. Our Lagrangian consists of two terms, $\mathcal{L}_{\rm wall}$ to produce a wall, and $\mathcal{L}_{\rm T}$ to obtain a coupling of tensor multiplets with the vector multiplet

$$\mathcal{L}_{\rm total} = \mathcal{L}_{\rm wall} + \mathcal{L}_{\rm T}\,. \tag{2.1}$$

As a concrete example, bosonic part of our Lagrangian for the wall reads

$$\begin{aligned}\mathcal{L}_{\rm wall}\big|_{\rm bosonic} = &-\frac{1}{4}F_{MN}(W)F^{MN}(W) + \frac{1}{2}\partial^M\Sigma\partial_M\Sigma + \mathcal{D}_M H^\dagger_{iA}\mathcal{D}^M H^{iA} - H^\dagger_{iA}(g_{\rm h}\Sigma - m_A)^2 H^{iA} \\ &+\frac{1}{2}(Y^a)^2 - g_{\rm h}\zeta^a Y^a + H^\dagger_{iA}(\sigma^a g_{\rm h} Y^a)^i_{\ j} H^{jA} + F^{\dagger i}_A F^A_i,\end{aligned} \tag{2.2}$$

where $g_{\rm h}$ denotes the hypermultiplet gauge coupling including its charge, $\mathcal{D}_M = \partial_M + ig_{\rm h}W_M$, and covariant derivative and ζ^a are the $SU(2)_R$ triplet of Fayet-Iliopoulos parameters[3]. Without loss of generality, we assume $\zeta^a = (0, 0, \zeta > 0)$, and $m_A = (m_{\rm h}, -m_{\rm h})$. It has been known for sometime that the above Lagrangian admits BPS single and multiple domain wall solutions in the limit of infinite gauge coupling, where the vector multiplet becomes just a Lagrange multiplier and the model reduces to a nonlinear sigma model with only hypermultiplets as physical degrees of freedom [15, 16]. However, we wish to retain the vector multiplet as a dynamical degree of freedom, rather than a Lagrange multiplier field. For that purpose, recently obtained exact solutions of BPS domain walls for discrete finite values of gauge coupling are extremely useful [6]. As the simplest case, we have shown that the exact solution of a single wall is obtained for a finite coupling

$$g_{\rm h}^2\zeta = 2m_{\rm h}^2. \tag{2.3}$$

Apart from the vicinity of the wall, the charged hypermultiplets takes nonvanishing value, and the $U(1)$ gauge symmetry is broken spontaneously in the bulk. The vector multiplet scalar Σ also exhibits a kink-like behavior interpolating between two vacua $\Sigma = \pm\frac{m_{\rm h}}{g_{\rm h}}$

$$\Sigma = \frac{m_{\rm h}}{g_{\rm h}}\tanh(m_{\rm h}y). \tag{2.4}$$

We find that the energy density is concentrated around $y = 0$ [6].

We wish to emphasize the general nature of our mechanism to localize a massless vector field. We only need a wall configuration for the scalar field Σ of a vector multiplet which couples to our tensor multiplets. Our explicit model for a wall is just to show that there is a consistent theory including all ingredients, in particular with SUSY. To emphasize this point, in the most part of this paper, we use only the following information on the background domain wall configuration : background value of the scalar field of the vector multiplet, $\langle\Sigma\rangle$, satisfies the BPS equation,

$$\langle\Sigma\rangle' \equiv \frac{d\langle\Sigma\rangle}{dy} = \langle Y^3\rangle, \quad \langle Y^1\rangle = \langle Y^2\rangle = 0, \tag{2.5}$$

where[4], $Y^{ij} = \sum_{a=1}^3 Y^a(i\sigma^a)^i{}_k\epsilon^{jk}$, and the $\langle\Sigma\rangle$ depends on only the coordinate y, and approaches two different values at left and right spacial infinity $y \to \pm\infty$, like the configuration (2.4). Furthermore, we assume the four-dimensional Lorentz invariance on the world volume, resulting in $\langle W_M\rangle = 0$. Whenever an explicit model becomes useful, we will always use our simplest exact solution in Eq.(2.4).

In an off-shell formulation of SUSY (and Supergravity) in five dimensions, two supermultiplet containing the antisymmetric tensor field B_{MN} can appear. One of the supermultiplets is called tensor gauge multiplet, whose B_{MN} is massless and admits gauge transformations by one-form, $\delta B_{MN} = 2\partial_{[M}\Lambda_{N]}$. The other supermultiplet is called the large (massive) tensor multiplet, whose bosonic components are antisymmetric tensor (two-form) field B^α_{MN}, scalar field σ^α, and $SU(2)_R$ triplet of auxiliary fields $X^\alpha_{ij} = X^\alpha_{ji}$, where $i, j = 1, 2$. They have to come in pairs, but we will see that one of the two can be interpreted as an auxiliary field. A pair of the large tensor multiplet $T^\alpha = (\sigma^\alpha,\, B^\alpha_{MN},\, X^{\alpha ij})$, $(\alpha = 1, 2)$ can have a mass term, and can carry a charge $g_{\rm t}$ for the $U(1)$ gauge field W_M. In our model, we use a pair of the large tensor multiplet as a minimal model. By

[3]We changed the normalization of the vector multiplet by g_h from Ref.[6] to make the kinetic term canonical.

[4]Our convention is $\epsilon_{12} = -\epsilon_{21} = \epsilon^{12} = -\epsilon^{21} = 1$. The $SU(2)_R$ indices are raised and lowered by contracting upper left with lower right indices as $Y^{ij} = \epsilon^{jk}Y^i{}_k = \epsilon^{ik}Y_k{}^j$.

now it is well-established that the most general Lagrangian for tensor and vector multiplets in five dimensions is characterized by a nonlinear kinetic term which is specified by second derivatives of a norm function $\mathcal{N}$ which is at most cubic in tensor and vector multiplets [17, 18, 19]. The invariance under the $U(1)$-gauge transformation with a gauge parameter Λ for this multiplet

$$\delta(\Lambda)T^\alpha = -\Lambda g_{\rm t}\epsilon_{\alpha\beta}T^\beta, \tag{2.6}$$

determines this cubic term of the norm function. This allows the tensor multiplets to interact with the vector multiplet, which carries informations of the domain wall configuration. By defining

$$\mathcal{M} \equiv m_{\rm t} - g_{\rm t}\Sigma, \tag{2.7}$$

we obtain the bosonic part of our Lagrangian containing tensor multiplets $\mathcal{L}_{\rm T}$ as

$$\begin{aligned}\mathcal{L}_{\rm T}|_{\rm bosonic} = \mathcal{M}\sum_{\alpha=1}^{2}\Big(&-\tfrac{1}{4}B^\alpha_{MN}B^{MN\alpha} + \tfrac{1}{2}\mathcal{D}^M\sigma^\alpha\mathcal{D}_M\sigma^\alpha + \tfrac{1}{4}X^{ij\alpha}X^\alpha_{ij} - \tfrac{1}{2}\mathcal{M}^2(\sigma^\alpha)^2\Big)\\ &- \sum_{\alpha=1}^{2}2g_{\rm t}\sigma^\alpha\left(\tfrac{1}{4}B^\alpha_{MN}F^{MN}(W) + \tfrac{1}{2}\mathcal{D}^M\sigma^\alpha\partial_M\Sigma + \tfrac{1}{4}X^\alpha_{ij}Y^{ij}\right)\\ &- \tfrac{1}{8}\epsilon^{MNLPQ}B^\alpha_{MN}\partial_L B^\beta_{PQ}\epsilon_{\alpha\beta} - \tfrac{1}{8}g_{\rm t}\epsilon^{MNLPQ}W_L B^\alpha_{MN}B^\alpha_{PQ},\end{aligned} \tag{2.8}$$

where, our convention of the space-time metric is $\eta_{MN} = \mathrm{diag}(1,-1,-1,-1,-1)$, and we omitted the auxiliary fields[5], which are not important for our model. The mass parameters $m_{\rm t}$ and the charge $g_{\rm t}$ are arbitrary at this point. However, we will later find that one of these two parameters must be tuned to assure the expected mechanism to work. The covariant derivative $\mathcal{D}_M\sigma^\alpha$ of the scalar field σ^α of the tensor multiplet is defined as usual

$$\mathcal{D}_M\sigma^\alpha = \partial_M\sigma^\alpha - g_{\rm t}W_M\epsilon_{\alpha\beta}\sigma^\beta. \tag{2.9}$$

The five-dimensional Lagrangian $\mathcal{L}_{\rm total} = \mathcal{L}_{\rm wall} + \mathcal{L}_{\rm T}$ given in (2.2) and (2.8) together with its fermionic terms are invariant under the five-dimensional SUSY transformation (with eight supercharges) . It contains kinetic terms of the 2-form tensor fields B_{MN}, as well as other fields.

Let us rewrite the Lagrangian (2.8) by integrating out the auxiliary field, X^α_{ij} and considering fluctuations around the vacuum expectation values. An equation of motion for the X^α_{ij} can be read as

$$\mathcal{M}\,X^\alpha_{ij} - g_{\rm t}\sigma^\alpha Y_{ij} = 0. \tag{2.10}$$

The equations of motion of the scalar σ^α are

$$0 = \mathcal{D}^M(\mathcal{M}\mathcal{D}_M\sigma^\alpha) + \sigma^\alpha\partial^M\partial_M\mathcal{M} + \mathcal{M}^3\sigma^\alpha + \tfrac{1}{2}g_{\rm t}B^\alpha_{MN}F_{MN}(W) + \tfrac{1}{2}g_{\rm t}X^\alpha_{ij}Y^{ij}. \tag{2.11}$$

These equations are consistent with

$$\langle\sigma^\alpha\rangle = \langle B^\alpha_{MN}\rangle = \langle X^\alpha_{ij}\rangle = 0, \tag{2.12}$$

which can also be derived from the requirement of the BPS condition preserving half of the eight SUSY. Namely the BPS wall solution (2.4) or (2.5) of the Lagrangian $\mathcal{L}_{\rm wall}$ is not disturbed by adding the tensor multiplet to the system.

[5] They are obtained as transformations by the central charge $Z\sigma^\alpha, Z^2\sigma^\alpha$ [19].

The quadratic terms of the fluctuations of the fields around the background (2.5), (2.12) can be read as

$$\begin{aligned}
\mathcal{L}_{\rm T}|_{\rm bosonic} = & \langle\mathcal{M}\rangle \sum_{\alpha=1}^{2}\left(-\frac{1}{4}B^{\alpha}_{MN}B^{MN\alpha}+\frac{1}{2}\partial^M\sigma^\alpha\partial_M\sigma^\alpha\right) \\
& -\sum_{\alpha=1}^{2}\frac{1}{2}\left(\langle\mathcal{M}\rangle^3+\frac{(\langle\mathcal{M}\rangle')^2}{\langle\mathcal{M}\rangle}-\langle\mathcal{M}\rangle''\right)(\sigma^\alpha)^2 \\
& -\frac{1}{8}\epsilon^{MNLPQ}B^{\alpha}_{MN}\partial_L B^{\beta}_{PQ}\epsilon_{\alpha\beta}+\sum_{\alpha=1}^{2}\left(-\frac{1}{2}\langle\mathcal{M}\rangle'(\sigma^\alpha)^2\right)' \\
& + (\text{higher order terms of the fluctuations}), \qquad (2.13)
\end{aligned}$$

where we used $\langle\mathcal{M}\rangle' = -g_{\rm t}\langle\Sigma\rangle' = -g_{\rm t}\langle Y^3\rangle$. We consider only this quadratic part of the Lagrangian in the following.

3 Massless Localized Vector Field from the Tensor Field

3.1 A Lagrangian for the 2-Form Tensor Fields

In this section, we concentrate on the kinetic term $\mathcal{L}'_{\rm 2form}$ for the 2-form tensor fields

$$\mathcal{L}'_{\rm 2form} = -\frac{1}{4}MB^1_{MN}B^{1MN}-\frac{1}{4}MB^2_{MN}B^{2MN}-\frac{1}{8}\epsilon^{LMNPQ}B^1_{MN}\partial_L B^2_{PQ}+\frac{1}{8}\epsilon^{LMNPQ}B^2_{MN}\partial_L B^1_{PQ} \qquad (3.1)$$

where we rewrite $M\equiv\langle\mathcal{M}\rangle(y)$ for simplicity. This Lagrangian is, so called, a self-dual Lagrangian in five dimensions. Furthermore, we rewrite the Lagrangian as follows

$$\begin{aligned}
\mathcal{L}'_{\rm 2form} &= \mathcal{L}_{\rm 2form}+\mathcal{L}_\theta, \\
\mathcal{L}_{\rm 2form} &= -\frac{1}{4}MB^1_{MN}B^{1MN}-\frac{1}{4}MB^2_{MN}B^{2MN}+\frac{1}{4}\epsilon^{LMNPQ}B^2_{MN}\partial_L B^1_{PQ}, \\
\mathcal{L}_\theta &= \partial_L\left(-\frac{1}{8}\epsilon^{LMNPQ}B^1_{MN}B^2_{PQ}\right). \qquad (3.2)
\end{aligned}$$

In a space extending to infinity without boundaries, we can freely use either one of these Lagrangians $\mathcal{L}'_{\rm 2form}$ and $\mathcal{L}_{\rm 2form}$, since total divergence term $\mathcal{L}_\theta$ does not contribute to the action. However, we are considering a wall soution which approaches to different vacua at left and right infinities, respectively, resulting in a topologically nontrivial configuration. Moreover, the $U(1)$ gauge invariance is spontaneously broken in the bulk. Therefore we need to decide how much total divergence terms should be included in our fundamental Lagrangian.

In order to extract physics out of our model, we note that one of the two-form tensor field should be treated as an auxiliary field. For instance, by varying with respect to B^2_{MN}, we obtain the equation of motion for B^2_{MN} as

$$0 = -MB^{2MN}+\frac{1}{2}\epsilon^{MNLPQ}\partial_L B^1_{PQ}, \qquad (3.3)$$

which allows to express B^2_{MN} algebraically in terms of B^1_{MN}. If we start from the Lagrangian $\mathcal{L}_{\rm 2form}$, we can derive the above equation without performing a partial integration. If we start from any other Lagrangian, such as $\mathcal{L}'_{\rm 2form}$ instead, we first need to add a total divergence term

$\mathcal{L}_\theta$ and obtain the Lgrangian $\mathcal{L}_{\rm 2form}$, so that we can derive the equation of motion (3.3). In order to interpret one of the tensor field B^2_{MN} as an auxiliary field, we decide to choose $\mathcal{L}_{\rm 2form}$ as our fundamental Lagrangian, and denote the remaining tensor field as $B_{MN} \equiv B^1_{MN}$ from now on.

If we consider a space-time with boundaries, for instance at $y = \pm\pi L$, applying the $U(1)$ gauge transformations (2.6) on our fundamental Lagrangian gives

$$\delta(\Lambda) \int_{-\pi L}^{\pi L} dy \int d^4x \mathcal{L}_{\rm 2form} = \int d^4x \left[g_{\rm t}\Lambda \frac{1}{8} \epsilon^{y\mu\nu\rho\sigma} (B^1_{\mu\nu} B^1_{\rho\sigma} - B^2_{\mu\nu} B^2_{\rho\sigma}) \right]_{-\pi L}^{\pi L}, \tag{3.4}$$

which vanishes if the fields are periodic ($B^1_{MN}(y = -\pi L) = B^1_{MN}(y = \pi L), B^2_{MN}(y = -\pi L) = B^2_{MN}(y = \pi L)$). For topologically nontrivial situations, fields are no longer periodic, and the $U(1)$ gauge invariance are broken on the boundaries [6] : the parameter of the transformation on the boundary must vanish on the boundaries ($\Lambda(\pi L) = \Lambda(-\pi L) = 0$). It is gauge invariant if there is no boundaries. If there is a boundary, the gauge degrees of freedom emerge as conformal field theories [21].

Since the Lagrangian $\mathcal{L}_{\rm 2form}$ is only quadratic in B^2_{MN}, we can perform the functional integral of B^2_{MN} exactly. The quadratic part of the resulting Lagrangian is now written in terms of $B_{MN} \equiv B^1_{MN}$ only

$$\mathcal{L}_{\rm 2form} = \frac{1}{12M(y)} F_{MNL}(B) F^{MNL}(B) - \tfrac{1}{4} M(y) B_{MN} B^{MN}. \tag{3.5}$$

This Lagrangian plays the most important role in our paper. If the mass function $M(y) = \langle \mathcal{M} \rangle = \langle m_{\rm t} - g_{\rm t}\Sigma(y) \rangle$ is a constant, this Lagrangian is reduced to an ordinary kinetic term for a massive tensor field, where no massless state is contained in the tensor field. Dubovsky and Rubakov observed, however, if one takes a limit of $M \to +0$ with $B_{MN}/\sqrt{M}$ fixed, the Lagrangian is reduced to the kinetic term for a 2-form gauge tensor field, that is, a massless field [14]. In our case, the scalar field $\Sigma(y)$ gives $M(y)$ a non-trivial dependence on y, which produces a region where $M(y)$ vanishes approximately. Therefore we can expect that the massless vector mode may exist in our system.

Now, let us examine this system in detail. By varying B_{MN}, we obtain the equation of motion of the physical field B_{MN} as

$$0 = \partial^L \left(\frac{1}{M(y)} F_{MNL}(B) \right) + M(y) B_{MN}. \tag{3.6}$$

To do partial integration here, a boundary condition is needed to eliminate the surface term

$$\int d^4x \left[\delta B^{\mu\nu} \left(\frac{1}{M(y)} F_{\mu\nu y}(B) \right) \right]_{y=-\infty}^{y=\infty} = 0. \tag{3.7}$$

We will study solutions of the equation (3.6) under the condition (3.7) in the next context.

3.2 Massless Modes and Localization

The most interesting and important point is the question whether the solution of the equation (3.6) contains a four-dimensional massless vector mode or not. Since we assume four-dimensional

[6] This is somewhat reminiscent of the Chern-Simons theory in three dimensions [20].

Lorentz invariance on the world volume, it is useful to introduce the momentum space $p_\mu, (\mu = 0, \cdots, 3)$ in four dimensions. Let us first study the massless modes $p^2 = 0$. In this case, it is useful to decompose four-dimensional Lorentz vectors in terms of the following basis vectors : the longitudinal component p_μ, the scalar component l_μ, and two transverse polarisation components $\epsilon^i_\mu, (i = 1, 2)$, which are defined by $l_\mu l^\mu = 0$, $l_\mu p^\mu = 1$, $\epsilon^i_\mu p^\mu = \epsilon^i_\mu l^\mu = 0$, and $\epsilon^i_\mu \epsilon^{\mu j} = -\delta^{ij}$. By substituting these expansions of the field B_{MN} to the equation (3.6), we obtain solutions as

$$\begin{aligned} B_{\mu\nu}(x,y)|_{\rm massless} &= \int \frac{d^4p}{(2\pi i)^4}\delta(p^2)\Big(2p_{[\mu}l_{\nu]}a(p)\phi_1(y) + 2ip_{[\mu}\epsilon^i_{\nu]}b_i(p)\phi_2(y) \\ &\qquad + 2il_{[\mu}\epsilon^i_{\nu]}c_i(p)\phi_3(y) + \epsilon^1_{[\mu}\epsilon^2_{\nu]}d(p)\phi_4(y)\Big)e^{ip_\lambda x^\lambda} \\ &\quad + \rho(y)\int \frac{d^4p}{(2\pi i)^4}\delta(p^2)2ip_{[\mu}\epsilon^i_{\nu]}c_i(p)e^{ip_\lambda x^\lambda}, \qquad (3.8) \\ B_{\mu y}(x,y)|_{\rm massless} &= \frac{1}{M(y)^2}\int \frac{d^4p}{(2\pi i)^4}\delta(p^2)\Big(ip_\mu a(p)\phi'_1(y) + \epsilon^i_\mu c_i(p)\phi'_3(y)\Big)e^{ip_\lambda x^\lambda}, \qquad (3.9) \end{aligned}$$

where, the four independent mode functions $\phi_a(y)$, $(a = 1, \cdots, 4)$ have to satisfy the same equation

$$0 = \left(\frac{\phi'(y)}{M(y)}\right)' - M(y)\phi(y), \qquad (3.10)$$

whereas another mode function $\rho(y)$ must be determined in terms of the mode function $\phi_3(y)$ as

$$0 = \left(\frac{\rho'(y)}{M(y)}\right)' - M(y)\rho(y) + \left(\frac{1}{M(y)^2}\right)'\frac{\phi'_3(y)}{M(y)}. \qquad (3.11)$$

If these mode functions are suitably normalizable, the corresponding four-dimensional fields $a(p), b_i(p), c_i(p), d(p)$ are physical massless fields.

If $\phi_3(y)$ happens to vanish, the function $\rho(y)$ satisfies the same equation as $\phi(y)$. Then there is no distinction between the field $c_i(p)$ and the field $b_i(p)$, and the term with $\rho(y)$ can be absorbed into the term with $b_i(p)$. Therefore we can define $\rho(y) = 0$ when $\phi_3(y) = 0$, and then the field $c_i(p)$ does not exist in that case.

The most general solution of Eq.(3.10) reads

$$\phi(y) = C_1 e^{s(y)} + C_2 e^{-s(y)}, \quad s(y) \equiv \int_y^\infty dz M(y). \qquad (3.12)$$

The behavior of this solution at $y \to \pm\infty$ shows that the scalar field configuration $M(y)$ must vanish at $y = \infty$ or $y = -\infty$. Otherwise, $\phi(y)$ (that is B_{MN}) diverges at $y = \infty$ or $y = -\infty$. Therefore, without loss of generality, we assume that

$$\begin{aligned} M(y) &\to +0, \quad \text{as} \quad y \to \infty, \qquad (3.13) \\ \phi(y) &= g_e^{-1}e^{-s(y)}, \quad (\phi' = M\phi). \qquad (3.14) \end{aligned}$$

where we denote the integration constant as g_e, since it will play a role of a coupling constant later. Note that the mode function $\phi(y)$ approaches the value g_e^{-1} asymptotically in the region where the background $M(y) = \langle m_{\rm t} - g_{\rm t}\Sigma(y)\rangle$ tends to vanish, whereas it vanishes at the opposite

infinity. On the other hand, we need not solve the equation for the $\rho(y)$, as we will find later that it does not provide physical normalizable mode.

To find out physical modes with normalizable wave functions, we will demand that the energy density T_{00} of the system to be bounded from above, since we will eventually consider continuum massive states as well. It is easiest to introduce spacetime metric g_{MN} temporarily and to take the flat space limit after varying the Lagrangian with respect to g_{MN}

$$T_{00} \equiv 2\frac{\delta S}{\delta g^{00}}\bigg|_{g^{MN}\to\eta^{MN}} = \left[\frac{1}{12M}F_{MNL}(B)^2 + \frac{M}{4}B_{MN}^2\right]\bigg|_{\eta^{MN}\to\delta^{MN}}. \tag{3.15}$$

The result of this manipulation is that the kinetic energy density of gauge fields (the first term of the right-hand side of Eq.(3.15)) is given by the sum of the square of the time derivatives $F_{0\mu\nu}$ (electric field) and the square of the spacial derivatives $F_{\mu\nu\lambda}$ (magnetic field) instead of their difference as in the Lagrangian. This point is represented by replacing η^{MN} by δ^{MN}. Therefore both the first and the second term of the right-hand side of this equation is positive definite. This formula contains the following term

$$\frac{M}{2}B_{\mu y}^2|_{\eta\to\delta} \tag{3.16}$$

and the contribution from the massless modes to this term has the following y-dependence

$$\frac{M}{2}\left(\frac{\phi'}{M^2}\right)^2 = \frac{\phi^2}{2M}, \tag{3.17}$$

where we used $\phi' = M\phi$, because of the solution (3.14). Clearly this term diverges at $y \to \infty$, showing the nonnormalizability of the mode. Hence, from the requirement of the bounded energy density, $c_i(p)$ and $a(p)$ components of the massless modes $B_{\mu y}$ in Eq.(3.9) should not exist as physical fields. Therefore, the part $B_{\mu y}$ of tensor field has no massless modes. Similarly, the following term contained in the energy density

$$\frac{1}{12M}F_{\mu\nu\lambda}(B)^2|_{\eta\to\delta} \tag{3.18}$$

has the same y-dependent contribution from the $d(p)$ component in Eq.(3.8). Therefore $F_{\mu\nu\lambda}(B)$ has no massless modes, that is, the $d(p)$ component field should vanish.

Therefore, up to this point, only the mode $b_i(p)$ remains as a candidate of normalizable massless mode of the system. This mode corresponds to the four-dimensional massless vector field $A_\mu(x)$ which we anticipated. Eq.(3.8) implies that the contribution of the component field $b_i(p)$ to the tensor field $B_{\mu\nu}$ can be expressed in terms of the field strength $F_{\mu\nu}(A) = 2\partial_{[\mu}A_{\nu]}$ of a vector potential A_μ and the function $\phi(y)$ as

$$B_{\mu\nu}(x,y)|_{\text{massless}} = \phi(y)F_{\mu\nu}(A(x)), \quad \partial^\nu F_{\mu\nu}(A) = 0, \tag{3.19}$$

$$B_{\mu y}(x,y)|_{\text{massless}} = 0. \tag{3.20}$$

Let us verify that our massless filed candidate $b_i(p)$ really gives a bounded energy density. The energy density (3.15) can now be given by a sum of the electric field $(\vec{E})_i = F_{0i}(A)$ and the magnetic field $(\vec{B})_i = \tilde{F}_{0i}(A)$ as

$$T_{00}(x,y) = M(y)\phi^2(y)\left(\vec{E}^2(x) + \vec{B}^2(x)\right) \equiv \frac{f(y)}{2{g_e}^2}\left(\vec{E}^2(x) + \vec{B}^2(x)\right), \tag{3.21}$$

where the profile $f(y)$ of the energy density is defined as

$$f(y) \equiv 2g_e^2 M(y)\phi(y)^2. \tag{3.22}$$

The effective four-dimensional gauge coupling g_e is defined by requiring the effective four-dimensional energy which is given by integrated over y to be

$$\int dy T_{00}(x,y) = \frac{1}{2g_e^2}\left(\vec{E}^2(x) + \vec{B}^2(x)\right). \tag{3.23}$$

The above result (3.21) shows that this effective four-dimensional gauge coupling g_e is a topological charge, which is determined solely by the boundary condition as

$$\int_{-\infty}^{\infty} dy 2M(y)\phi(y)^2 = \int_{-\infty}^{\infty} dy \frac{d}{dy}\left(\phi(y)^2\right) = \left[\phi(y)^2\right]_{-\infty}^{\infty} = \frac{1}{g_e^2}, \tag{3.24}$$

where we used the equation (3.14). Thus the profile function of the energy density is normalized as

$$\int_{-\infty}^{\infty} dy f(y) = 1. \tag{3.25}$$

We can easily find that the configuration of $f(y)$ vanishes at $y \to \infty$ because of $M(y) \to 0$, and also vanishes at $y \to -\infty$ because of $\phi(y) \to 0$, and the region corresponding to the wall, where the configuration of $M(y) \equiv \langle \mathcal{M} \rangle = \langle m_{\rm t} - g_{\rm t}\Sigma \rangle$ varies, gives a finite contribution of $f(y)$. This behavior of the energy density illustrate that *the four-dimensional massless vector field A_μ is localized on the wall as the solution of the equations of motion for the tensor multiplets.*

To illustrate this localization mechanism by an explicit solution of our wall, we can take the exact solution (2.4) as an example. Since we must satisfy the condition of vanishing $M(y) = \langle m_{\rm t} - g_{\rm t}\Sigma(y) \rangle$ as $y \to \infty$ (3.13), we require

$$0 = m_{\rm t} - g_{\rm t} m_{\rm h}/g_{\rm h}. \tag{3.26}$$

We will use λ defined by $m_{\rm t}/m_{\rm h} = g_{\rm t}/g_{\rm h} \equiv \lambda/2 > 0$ together with $m_{\rm h}$ as the two independent parameters. We obtain

$$M(y) \equiv m_{\rm t} - g_{\rm t}\langle \Sigma \rangle = \frac{\lambda m_{\rm h}}{2}(1 - \tanh(m_{\rm h} y)) = \frac{\lambda m_{\rm h} e^{-m_{\rm h} y}}{2\cosh(m_{\rm h} y)}. \tag{3.27}$$

The mode function $\phi(y)$ and the energy density profile $f(y)$ are given by

$$\phi(y) = g_e^{-1}\left(\frac{e^{m_{\rm h} y}}{2\cosh(m_{\rm h} y)}\right)^{\frac{\lambda}{2}}, \tag{3.28}$$

$$f(y) = \frac{\lambda m_{\rm h} e^{(\lambda-1)m_{\rm h} y}}{2^\lambda \cosh^{(1+\lambda)}(m_{\rm h} y)}, \quad \left\{ \begin{array}{lll} f(y) & \to \ \frac{\lambda m_{\rm h}}{2} e^{-2m_{\rm h} y} & (y \to \infty) \\ f(y) & \to \ \frac{\lambda m_{\rm h}}{2} e^{2\lambda m_{\rm h} y} & (y \to -\infty) \end{array} \right. . \tag{3.29}$$

The profile of these functions in Eqs.(3.28) and (3.29) are illustrated in Fig.1 and Fig.2, respectively.

To close this subsection, we have to verify that our solution satisfies the boundary condition (3.7), since the mode function $\phi(y)$ does not vanish on the boundary. we can easily confirm the validity, by noting

$$\delta B_{\mu\nu}\Big|_{\rm bondary} = 2\partial_{[\mu}\delta A_{\nu]}, \tag{3.30}$$

and by performing a four-dimensional partial integration.

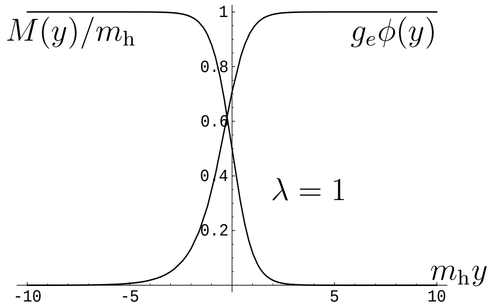

Figure 1: The mass function $M(y)$ in Eq.(3.27) and the mode function $\phi(y)$ in Eq.(3.28).

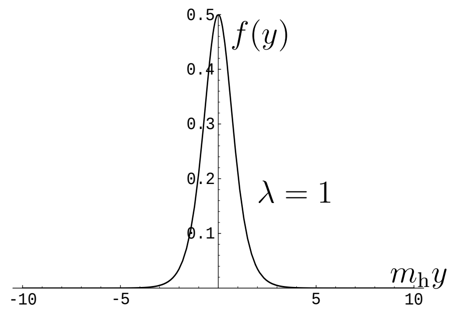

Figure 2: The profile function $f(y)$ of the energy density in Eq.(3.29).

3.3 Mass Spectrum and Four-Dimensional Effective Lagrangian

To obtain not only low-energy effective Lagrangian, but also the entire action on the background of our wall solution, it is necessary to work out all the massive modes. The result may also be of use in future study of the system. The basis vectors in momentum space of the massive states are three polarization vectors ϵ^i_μ with $i = 1, \cdots, 3$ defined by $p^\mu \epsilon^i_\mu = 0$ together with the momentum p^μ itself. We substitute momentum-expansions of the fields B_{MN} to the equation (3.6) with the assumption $p^2 \neq 0$ to obtain

$$B_{\mu\nu}(x,y)|_{\rm massive} = \sum_{n\geq 1} \int \frac{d^4p}{(2\pi i)^4} \delta(p^2 - m_{(n)}^2) e^{ip_\lambda x^\lambda} \left\{ 2ip_{[\mu}\epsilon^i_{\nu]} c_i(p) \frac{u'_{(n)}(y)}{M(y)} + \epsilon^i_\mu \epsilon^j_n d_{ij}(p) u_{(n)}(y) \right\}, \tag{3.31}$$

$$B_{\mu y}(x,y)|_{\rm massive} = \sum_{n\geq 1} \int \frac{d^4p}{(2\pi i)^4} \delta(p^2 - m_{(n)}^2) m_{(n)}^2 e^{ip_\lambda x^\lambda} \epsilon^i_\mu c_i(p) \frac{u_{(n)}(y)}{M(y)}, \tag{3.32}$$

where the function $u_{(n)}(y)$ is an eigenfunction of the following eigenvalue equation with an eigenvalue $m_{(n)}$, which serves as mass squared of the associated four-dimensional field

$$\begin{aligned} Ku_{(n)} &= m_{(n)}^2 u_{(n)}, \quad n = 1, 2, \cdots \\ K &\equiv -M(y) \frac{d}{dy} \frac{1}{M(y)} \frac{d}{dy} + M(y)^2. \end{aligned} \tag{3.33}$$

Note that the function $\phi(y)$ for the massless mode can be identified as the eigenfunction for zero eigenvalue : $K\phi = 0$.

Let us also consider modes of the scalars σ^α of the tensor multiplets. The linearised equation

of motion for the scalars σ^α can be read from the Lagrangian (2.13)

$$0 = M(y)\partial^\mu\partial_\mu\sigma^\alpha - \left(M(y)\sigma^{\alpha\prime}\right)' - M(y)''\sigma^\alpha + M(y)^3\sigma^\alpha + \frac{\left(M(y)'\right)^2}{M(y)}\sigma^\alpha. \tag{3.34}$$

A similar argument using energy density shows that there is no massless modes in σ^α. For massive modes, this equation can also be solved by using the function $u_{(n)}(y)$

$$\sigma^\alpha(x,y) = \sum_n \int \frac{d^4p}{(2\pi i)^4}\delta(p^2 - m_{(n)}^2)e^{ip_\lambda x^\lambda}\tilde{\sigma}^\alpha(p)\frac{u_{(n)}(y)}{M(y)}. \tag{3.35}$$

This fact may be a result of the unbroken $D = 4, \mathcal{N} = 1$ SUSY.

To obtain a mass spectrum of the system, let us discuss the normalization and boundary conditions for massive state. In calculating the energy density of the system in four dimensions, we encounter the following quantities,

$$(u_{(n)},\, u_{(m)}), \quad (\phi,\, u_{(n)}), \quad (u_{(n)},\, Ku_{(m)}), \cdots, \tag{3.36}$$

where the inner product $(u,\, v)$ is defined by,

$$(u,\, v) \equiv \int_{-\pi L}^{\pi L} dy \frac{u(y)v(y)}{M(y)}. \tag{3.37}$$

Since we eventually need to treat continuum of states, we will assume for reguralization purposes a compact space for the extra dimension y, as $-\pi L \le y \le \pi L$, and we will take the limit of $L \to \infty$ in the final stage. Therefore, the normalization of the function $u_{(n)}$ must be defined by means of the inner product $(u_{(n)},\, u_{(m)})$. This inner product involving the operator K in Eq.(3.33) has the following property

$$(u, Kv) = \Delta(u,v) + (Ku, v), \qquad \Delta[u,v] \equiv \left[-\frac{uv'}{M} + \frac{u'v}{M}\right]_{-\pi L}^{\pi L}. \tag{3.38}$$

To make the operator K hermitian $(u, Kv) = (Ku, v)$ with respect to the inner product (3.37), we should demand that the contribution from the boundary, $\Delta[u,v]$, must vanish

$$\Delta[u_{(n)}, u_{(m)}] = \left[-\frac{u_{(n)}u'_{(m)}}{M} + \frac{u'_{(n)}u_{(m)}}{M}\right]_{-\pi L}^{\pi L} = 0, \tag{3.39}$$

$$\Delta[\phi, u_{(n)}] = \left[-\phi\left(\frac{u'_{(n)}}{M} - u_{(n)}\right)\right]_{-\pi L}^{\pi L} = 0. \tag{3.40}$$

Eq.(3.40) means that the massless mode is orthogonal to the massive modes. This is satisfied if the boundary conditions of the eigenfunction $u_{(n)}$ are given by

$$\mathcal{B}_{(n)}(\pi L) = \mathcal{B}_{(n)}(-\pi L) = 0, \quad \mathcal{B}_{(n)}(y) \equiv u'_{(n)}(y) - M(y)u_{(n)}(y). \tag{3.41}$$

These boundary conditions are also enough to satisfy Eq.(3.39). With these conditions, the inner products of the eigenfunctions can be normalised as

$$(u_{(n)}, u_{(m)}) = \delta_{nm}, \quad (\phi, u_{(n)}) = 0. \tag{3.42}$$

On the other hand, the normalization of the massless mode function $\phi(y)$ is performed by

$$\int_{-\pi L}^{\pi L} dy M\phi^2 = \frac{1}{2}\left[\phi^2\right]_{-\pi L}^{\pi L} \equiv \frac{1}{2g_e^2}, \tag{3.43}$$

which we found in Sect.3.2 as the contribution from the massless mode to the energy density. In fact, we find a divergent result $(\phi, \phi) = \infty$, if we apply the inner product (3.37) blindly also to the massless mode. We never encounter this quantity in the calculating the energy density. With these normalization and the boundary conditions, we can compute the mass spectrum by a numerical analysis once the quantity $M(y) = \langle m_{\rm t} - g_{\rm t}\Sigma(y)\rangle$ is given. In the case of our exact solution (3.27) with $\lambda = 1$, we can solve the equation (3.33) exactly and find the exact mass spectrum

$$m_{(n)} = \sqrt{m_{\rm h}^2 + \left(\frac{n}{2L}\right)^2}, \qquad n = 1, 2, \cdots, \tag{3.44}$$

which will be derived in the Appendix. In this case, we can explicitly see that the massless mode is always isolated from the massive mode even in the limit of $L \to \infty$, because of the mass gap. We expect that this desirable property will persist with other values of couplings and other configurations of the $M(y)$.

Let us also describe the four-dimensional effective Lagrangian of the system to the second order of the fluctuations. Assuming that above mode functions $\phi(y), u_{(n)}(y)$ form a complete set to expand a function of y, we obtain expansions of the tensor fields B_{MN} and the scalar fields σ^α as

$$B_{\mu\nu}(x, y) = \phi(y)F_{\mu\nu}(A(x)) + \sum_{n\geq 1}\left(\frac{u'_{(n)}(y)}{m_{(n)}M(y)}F_{\mu\nu}(A^{(n)}(x)) + u_{(n)}(y)C_{\mu\nu}^{(n)}(x)\right), \tag{3.45}$$

$$B_{\mu y}(x, y) = \sum_{n\geq 1} m_{(n)}\frac{u_{(n)}(y)}{M(y)}A_\mu^{(n)}(x), \tag{3.46}$$

$$\sigma^\alpha(x, y) = \sum_{n\geq 1}\frac{u_{(n)}(y)}{M(y)}\tilde{\sigma}_{(n)}^\alpha(x). \tag{3.47}$$

Substituting this expansion to the Lagrangian, the quadratic terms of the four-dimensional effective Lagrangian can be read as

$$\mathcal{L}^{\rm eff} = \mathcal{L}_{\rm boundary} + \sum_{n\geq 1}\mathcal{L}_{\rm massive}^{(n)} + \mathcal{L}_{\rm interactions}. \tag{3.48}$$

The first term is a contribution from the boundaries

$$\mathcal{L}_{\rm boundary} = -\frac{1}{4}\left[\left\{\phi F_{\mu\nu}(A) + \sum_{n\geq 1}\left(\frac{u_{(n)}}{m_{(n)}}F_{\mu\nu}(A^{(n)}) + u_{(n)}C_{\mu\nu}^{(n)}\right)\right\}^2\right]_{-\pi L}^{\pi L}, \tag{3.49}$$

where we used the boundary conditions. The second term corresponds to the kinetic term for massive states

$$\begin{aligned}\mathcal{L}_{\rm massive}^{(n)} = &-\frac{1}{4}F_{\mu\nu}(A^{(n)})^2 + \frac{m_{(n)}^2}{2}(A_\mu^{(n)})^2 + \frac{1}{12}F_{\mu\nu\lambda}(C^{(n)})^2 - \frac{m_{(n)}^2}{4}(C_{\mu\nu}^{(n)})^2 \\ &+ \frac{1}{2}\partial^\mu\tilde{\sigma}_{(n)}^\alpha\partial_\mu\tilde{\sigma}_{(n)}^\alpha - \frac{m_{(n)}^2}{2}(\tilde{\sigma}_{(n)}^\alpha)^2.\end{aligned} \tag{3.50}$$

The $\mathcal{L}_{\text{interactions}}$ contains interactions which we do not consider here. If we take a limit of $L \to \infty$, we find that the term $\mathcal{L}_{\text{boundary}}$ reduces to the kinetic term of the massless vector fields

$$\mathcal{L}_{\text{boundary}} \quad \to \quad \mathcal{L}_{\text{massless}} = -\frac{1}{4g_e^2} F_{\mu\nu}(A)^2, \tag{3.51}$$

where, we used $u_{(n)}(y) \sim 1/\sqrt{L}$ at $y = \pm\pi L$ and thus, the values of $u_{(n)}$ on the boundaries vanish in this limit.

4 Four-Dimensional Coulomb Law

As we explained in Sect.3, we demonstrated that the four-dimensional massless vector field A_μ generated from the solution of the tensor field B_{MN} is localized on the BPS domain wall in this system. It is interesting and important to identify particles carrying the charge for the gauge transformation $\delta A_\mu = -\partial_\mu \Lambda$ of this massless localized vector field, which is not available in our present system. We expect that the gauge field $A_\mu(x)$ may be similar to the electromagnetic dual of the fundamental vector field W_M in our model. The fundamental vector field W_M is the gauge field of the other $U(1)$ gauge transformation (2.6) broken by the wall solution. Thjerefore we shall call the source for the fundamental vector field W_M as "electric" and the source associated to the tensor field B_{MN} as magnetic. In Ref.[13], the model is embedded into an $\mathcal{N} = 2$ SUSY $SU(2)$ gauge theory in four dimensions from the beginning. Therefore they were able to identify the source of the magnetic charge by incorporating classical solutions such as the Abrikosov-Nielsen-Olesen magnetic flux tube. In this way, they were able to show that the flux carried by the Abrikosov-Nielsen-Olesen magnetic flux tube becomes the source of the massless localized vector field that they found. In a similar spirit, it is an interesting and challenging task to generalize our model to non-Abelian gauge group such that the source of our massless localized vector field may be constructed as a classical solution, such as monopoles. Since we have not yet succeeded in building such a generalization, we will show only that our massless localized vector field does exhibit the four-dimensional Coulomb law between sources which we introduce here as external sources.

Let us introduce a source $\mathcal{T}^{MN}(x, y)$ for the tensor field B_{MN}

$$\mathcal{L} = \frac{1}{12M} F_{MNL}(B) F^{MNL}(B) - \frac{M}{4} B_{MN} B^{MN} + \tfrac{1}{2} B_{MN} \mathcal{T}^{MN}. \tag{4.1}$$

The equation of motion for the tensor field now reads

$$\partial^L \left(\frac{1}{M} F_{MNL}(B) \right) + M B_{MN} = \mathcal{T}_{MN}. \tag{4.2}$$

By taking divergence, we obtain a source corresponding to the magnetic charge current $\tilde{J}^M(x, y)$ which is now introduced as an external source [14]

$$\partial_N (M B^{MN}) = \partial_N \mathcal{T}^{MN} \equiv -\tilde{J}^M, \quad \partial_M \tilde{J}^M = 0. \tag{4.3}$$

If we introduce the magnetic source $\tilde{J}^M(x, y)$ on the wall near $y = 0$, but not in the bulk, it is reasonable to assume that the background is not disturbed by the source, and that only the massless mode is excited as in Eq.(3.19)

$$B_{\mu\nu}(x, y) = \phi(y) F_{\mu\nu}(A(x)), \quad B_{\mu y}(x, y) = 0. \tag{4.4}$$

The corresponding source $\mathcal{T}_{MN}$ for the tensor field can be read from the equation of motion (4.2)

$$\mathcal{T}_{\mu\nu}(x,y)=0, \quad \mathcal{T}_{\mu y}(x,y)=\phi(y)\partial^\nu F_{\nu\mu}(A(x)). \tag{4.5}$$

Then Eq.(4.3) implies the following distribution of the magnetic charge current $\tilde{J}_M(x,y)$

$$\tilde{J}_\mu(x,y)=\phi'(y)\partial^\nu F_{\nu\mu}(A), \quad \tilde{J}_4(x,y)=\partial^\nu \mathcal{T}_{\nu y}=0. \tag{4.6}$$

Now we can view the first equation as the usual equation for the source exciting our massless gauge field

$$\partial^\nu F_{\mu\nu}(A(x))=-g_e^2 J_\mu(x), \tag{4.7}$$

where the source current for our gauge field $J_\mu(x)$ is defined in terms of the magnetic source current $\tilde{J}_M(x,y)$ as

$$\tilde{J}_\mu(x,y)=g_e^2\phi'(y)J_\mu(x), \quad \tilde{J}_4(x,y)=0. \tag{4.8}$$

We can see that this configuration is consistent with the precondition of putting the magnetic source on the wall, since the function $\phi'(y)=M(y)\phi(y)$ is localized on the wall. If we take a static point source for the massless localized vector field $J_\mu(x)=\delta^0_\mu\delta^3(x)$, we can easily see from the above Eq.(4.7) that four-dimensional Coulomb law of the usual minimal electromagnetic interaction is reproduced.

On the other hand, the source $\mathcal{T}_{MN}(x,y)$ for the tensor multiplet which causes this configuration is given by

$$\mathcal{T}_{\mu\nu}(x,y)=0, \quad \mathcal{T}_{\mu y}(x,y)=g_e^2\phi(y)J_\mu(x). \tag{4.9}$$

It is interesting to note the field $\phi(y)$ approaches a nonvanishing constant value asymptotically as illustrated in Fig.1. This behavior of the tensor field source $\mathcal{T}_{\mu y}(x,y)$ appears to suggest a certain flux coming out of the brane to positive infinity $y=\infty$. However, we believe that this should be a fictitious flux like a Dirac string for a monopole, since the energy density corresponding to the massless vector excitation is localized around the wall, as shown in Eq.(3.21). If we wish to place the magnetic source $\tilde{J}^M(x,y)$ in the bulk, we need to take into account of the deformation of the background due to the presence of the magnetic source [14]. We wish to investigate the nature of the massless vector field and its coupling further in subsequent publications.

5 Discussion

We can extend our mechanism for a massless localized gauge field on a wall to an arbitrary space-time dimensions, provided we ignore SUSY for the moment. Suppose that we have the same bosonic Lagrangian as our Lagrangian $\mathcal{L}_{\rm wall}|_{\rm bosonic}$ in (2.2) to build a wall in arbitrary D space-time dimensions. Let us add the following Lagrangian for a $(D-3)$-form field $B_{\mu_1\cdots\mu_{D-3}}$ instead of Eq.(3.5) in five dimensions

$$\mathcal{L}_{(D-3){\rm form}}=\frac{1}{2(D-2)!M}F_{\mu_1\cdots\mu_{D-2}}(B)F^{\mu_1\cdots\mu_{D-2}}(B)-\frac{M}{2(D-3)!}B_{\mu_1\cdots\mu_{D-3}}B^{\mu_1\cdots\mu_{D-3}}, \tag{5.1}$$

$$\mathcal{L}_{\rm total}=\mathcal{L}_{\rm wall}|_{\rm bosonic}+\mathcal{L}_{(D-3){\rm form}}. \tag{5.2}$$

We expect that the same mechanism may be operative in this system as well: namely a massless localized $D-4$ form field $A_{\mu_1\cdots\mu_{D-4}}$ is likely to be contained in the $D-3$ form field $B_{\mu_1\cdots\mu_{D-3}}$ as $B_{\mu_1\cdots\mu_{D-3}}(x,y) = \phi(y)(D-3)\partial_{[\mu_1}A_{\mu_2\cdots\mu_{D-3}]}(x)$, where $\phi(y)$ is a mode function of the massless form field. However, we should note that it may or may not be realized with SUSY, since the constraint of SUSY in higher dimensions are quite severe. We can think of the above Lagrangian just a bosonic model without SUSY, although it is motivated from SUSY models. The electromagnetic dual field of the fundamental vector field W_M in D dimensions should be a $D-3$ form, and the electromagnetic dual of W_M in $D-1$ dimensions should be a $D-4$ form. Therefore the fundamental $D-3$ form field B and its massless localized component of $D-4$ form field A precisely possesses the expected degree of forms.

Let us finally list some of open problems for future research.

It is most desirable to be able to obtain charged fields which interact with our massless localized gauge field. This may be achieved by introducing a non-Abelian generalization of our model. Therefore it is an interesting open problem to make our model non-Abelian, such as $SU(2)$. This might answer the question whether our massless localized gauge field is really an electromagnetic dual of the fundamental gauge field. It may hopefully lead to more realistic model building with the $\mathcal{N}=1$ SUSY standard model matter content [23].

It would be interesting to understand more deeply the symmetry or topological reason for the existence of massless localized gauge field.

We had to make one fine-tuning among parameters of the hypermultiplet and tensor multiplet (3.13) or (3.26) to obtain a massless localized gauge field. It is also an interesting open question to understand or explain this fine-tuning from other argument. If this question can be addressed successfully, it may also be possible to fix other parameters of our model, such as $\lambda/2 \equiv g_t/g_{\rm h} = m_{\rm t}/m_{\rm h}$.

It should be straight-forward to embed our system into supergravity in five dimensions [19, 16, 22]. Then there are of course interesting questions to be explored, such as the fate of graviphoton.

Although we have obtained the four-dimensional gauge coupling as a topological charge, it can still be compatible with the concept of running coupling due to quantum effects. We can draw an interesting analogy to the fact that the masses of the BPS magnetic monopole and dyon are characterized as topological charge, which appear as the central charge in the SUSY algebra. The exact solution of the $\mathcal{N}=2$ SUSY gauge theories [24] demonstrated explicitly that these topological charges receive interesting nonperturbative effects from quantum loops. It is a challenging future problem to consider quantum effects in our theory.

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A Massive modes

Let us consider the solution of the equation of motion for the massive mode (3.33) with the boundary condition (3.41). A canonical mode functions $v_{(n)}(y) \equiv u_{(n)}(y)/\sqrt{M}$ may be more convenient than the original mode functions $u_{(n)}(y)$, because of the definition of the inner product (3.37). Rewriting the equation of motion (3.33) by the canonical mode functions $v_{(n)}(y)$, we obtain an ordinary Schrödinger equation

$$\left(-\frac{d^2}{dy^2} + V(y)\right) v_{(n)}(y) = m_{(n)}^2 v_{(n)}(y), \tag{A.1}$$

where the potential $V(y)$ is given by

$$V(y) = M(y)^2 - \sqrt{M(y)} \left(\frac{\left(\sqrt{M(y)}\right)'}{M(y)}\right)'. \tag{A.2}$$

If we use the configuration (3.27) for the quantity $M(y)$, the $V(y)$ can be read as

$$V(y) = \frac{m_{\rm h}^2}{4\cosh^2(m_{\rm h} y)} \left(2 + (1+\lambda^2)\cosh(2m_{\rm h} y) + (1-\lambda^2)\sinh(2m_{\rm h} y)\right). \tag{A.3}$$

The potential $V(y)$ as a function of y approaches the value $m_{\rm h}^2$ asymptotically at $y \to \infty$, and approaches the value $\lambda^2 m_{\rm h}^2$ at the opposite infinity. Therefore, we find that the mass gap between the massless mode and the massive mode is $\lambda m_{\rm h}$ for the case $\lambda \leq 1$, whereas the mass gap is $m_{\rm h}$ for the case $\lambda > 1$.

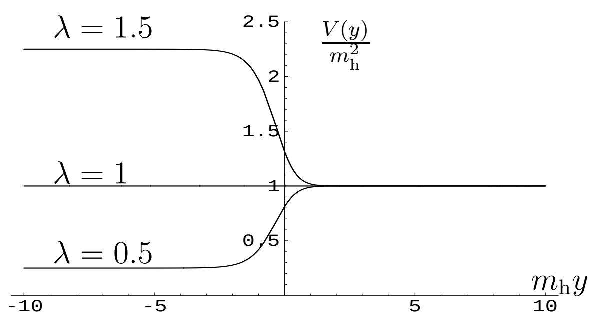

Figure 3: The potential $V(y)$ as a function of y for $\lambda = 0.5, 1, 1.5$.

We note that the potential becomes a constant in the case of $\lambda = 1$: $V(y) = m_{\rm h}^2$. Therefore we can fortunately solve the equation of motion exactly and obtain the mass spectrum for $\lambda = 1$. In this case, the mass function $M(y) = \langle m_{\rm t} - g_{\rm t}\Sigma(y)\rangle$ is given by the configuration of the vector multiplet scalar Σ in Eq.(2.4) as

$$M(y) = \frac{m_{\rm h}}{2}\left\{1 - \tanh(m_{\rm h}(y - y_0))\right\} > 0, \tag{A.4}$$

where we have restored an arbitrary parameter y_0 corresponding to the position of the wall. Exact solutions of the equation of motion (3.33) are given by

$$\begin{aligned} u_{(0)}(y) \equiv \phi(y) &= \frac{e^{\frac{m_{\rm h}}{2}(y-y_0)}}{g_e(L)\sqrt{2\cosh(m_{\rm h}(y-y_0))}} = \sqrt{\frac{M(y)}{m_{\rm h} g_e^2(L)}}\, e^{m_{\rm h}(y-y_0)}, \quad m_0 = 0, \\ u_{(n)}(y) &= C_{(n)}\sqrt{M(y)}\cos(\tilde{m}_{(n)}(y - y_{(n)})), \quad m_{(n)} = \sqrt{m_{\rm h}^2 + \tilde{m}_{(n)}^2}, \quad n = 1, 2, \cdots, \end{aligned} \tag{A.5}$$

where a parameter $g_e(L)$ is defined by the normalization (3.43) and reduces to the effective four-dimensional gauge coupling g_e in the limit $L \to \infty$, and $y_{(n)}$, $\tilde{m}_{(n)}$ are arbitrary constant parameters. In this case, the quantity $\mathcal{B}_{(n)}(y)$ defined by Eq.(3.41) is given by

$$\mathcal{B}_{(n)}(y) = -C_{(n)}\sqrt{M(y)}\left(m_{\rm h}\cos(\tilde{m}_{(n)}(y-y_{(n)})) + \tilde{m}_{(n)}\sin(\tilde{m}_{(n)}(y-y_{(n)}))\right). \tag{A.6}$$

Thus, the boundary conditions (3.41) become

$$\tan(\tilde{m}_{(n)}(\pi L - y_{(n)})) = -\frac{m_{\rm h}}{\tilde{m}_{(n)}}, \qquad \tan(\tilde{m}_{(n)}(\pi L + y_{(n)})) = \frac{m_{\rm h}}{\tilde{m}_{(n)}}, \tag{A.7}$$

which determine the parameters $y_{(n)}$, $\tilde{m}_{(n)}$ as

$$\tilde{m}_{(n)} = \frac{n}{2L}, \qquad y_{(n)} = L\left(\frac{2}{n}\arctan\left(\frac{2m_{\rm h}L}{n}\right) - \pi\right), \qquad n = 1, 2, \cdots. \tag{A.8}$$

Note that a massive mode corresponding to $n = 0$ is prohibited by the boundary condition, whereas the massless mode is permitted. Therefore the mass spectrum can be read as

$$m_{(n)} = \sqrt{m_{\rm h}^2 + \left(\frac{n}{2L}\right)^2}, \qquad n = 1, 2, \cdots \tag{A.9}$$

The normalization constants are found to be

$$1 = \int_{-\pi L}^{\pi L} dy \frac{u_{(n)}(y)^2}{M(y)} = C_{(n)}^2 \int_{-\pi L}^{\pi L} dy' \cos^2\left(\frac{n\, y'}{2L}\right) = \pi L C_{(n)}^2. \tag{A.10}$$

Thus we obtain the exact solutions for the massive modes

$$\begin{aligned} u_{2n}(y) &= \sqrt{\frac{M(y)}{\pi L}}\cos\left(\frac{n\, y}{L} - \arctan\left(\frac{m_{\rm h}L}{n}\right)\right), \qquad n = 1, 2, 3, \cdots \\ u_{2n+1}(y) &= \sqrt{\frac{M(y)}{\pi L}}\sin\left(\frac{(2n+1)y}{2L} - \arctan\left(\frac{2m_{\rm h}L}{2n+1}\right)\right), \qquad n = 1, 2, 3, \cdots. \end{aligned} \tag{A.11}$$

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